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QUANTUM MECHANICS AS A THEORY OF RELATIVISTIC BROWNIAN MOTION

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ABSTRACT. It is shown that nonrelativistic quantum mechanics can be discussed as a variant of the relativistic statistical theory which describes the random motion of a classical particle. The relativity of the theory consists of the fact that a relativistic concept of the particle's state is used in describing the particle's behavior. This situation is fundamental because the statistics are the calculation of states and the result depends on what is designated a state.

The magnificent structure of contemporary quantum mechanics stands as an isolated phenomenon. A connection between quantum and classical mechanics is accomplished only with the help of the correspondence principle. It seems that quantum mechanics represents a fundamentally different theory [1,2] which cannot be understood from the standpoint of classical mechanics. Attempts of various authors [3-12] to understand quantum mechanics from the standpoint of classical mechanics have not fully achieved the goal.[2] Although this sounds paradoxical, the reason for this consists, in our opinion, of the fact that they have attempted to understand nonrelativistic quantum mechanics from the standpoint of nonrelativistic classical mechanics. /5*

We will show in the present communication that quantum mechanics represents a variant of the relativistic theory of Brownian motion. The difference from the approach of the other authors consists of the fact that we discuss <u>nonrelativistic</u> quantum mechanics from the point of view of <u>relativistic</u> classical mechanics. With this approach no new principles are required to understand the

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2. The reference [13] is an exception. Here success is achieved, as it seems to us, by not taking clear account of what we call the relativistic concept of a state.

*Numbers in the margin indicate pagination in the foreign text.

quantum mechanics of a single particle. In particular, such specifically quantum principles as the uncertainty principle and the correspondence principle can be understood from the classical viewpoint. The key to such an understanding is a relativistic concept of the state of a system. /6

1. The Concept of a State

In nonrelativistic physics one understands by the state of a physical system the set of quantities specified at a certain instant and possessing the property that being specified at one time they permit the determination of these quantities (i.e., the state) at all succeeding times. The equations of motion, which describe the evolution of the state in time, serve for this purpose. The state is described for a system consisting of particles by specifying the coordinates and momenta at a specified time; for a system consisting of a liquid or a gas, the state is determined by specifying the velocities of all the elements of the liquid, the density, temperature, and so forth. The state and equations of motion which describe the temporal evolution of the state represent two necessary elements of any nonrelativistic physical theory. The connection between these two elements is purely superficial.

As follows from the definition, the state of a system is specified at a specified time. But in relativistic theory simultaneity is relative, and which events are to be considered simultaneous and which not depends mainly on the choice of a system of coordinates, i.e., on our arbitrariness. If, for example, the state of a physical system is known to us in the system of coordinates, K, /7
then we can determine the state in the system of coordinates K', which is moving with respect to the system of coordinates K, only in that case where the equations of motion are known and we know how to solve them. Actually, the state in the system of coordinates K is specified on the hyperplane t= const in the four-dimensional events space but the state is specified in the system of coordinates K' on a different hyperplane t' = const, and the conversion of the state from one hyperplane to the other is possible only in the case in which the equations of motion are known. Thus in the relativistic theory the state and the equations of motion are closely connected. Because of the absence of absolute simultaneity in the relativistic theory it seems more consistent to define the state of a system not at a given time but at once in the entire events space. The concept

of a state includes within it the law of temporal evolution of the physical system. The equations of motion now take on another meaning. We will discuss them as certain limitations which are imposed on the possible states. Of all the possible states not all but only those which satisfy the equations are realized. We will call them the coupling equations. Essentially these are the same equations of motion, but now they describe not the evolution of the state in time but are limitations which isolate the physically-permissible states from the virtual ones.

The situation recalls statics, where there is no talk about the evolution of the states but the states are divided into two classes: permissible (equilibrium) and inadmissible (nonequilibrium). We will thus talk about "statics in events space".

The ideas presented above can be concisely formulated in the following /8
manner. In nonrelativistic theory the unique division into states and equations of motion in the description of physical phenomena corresponds in the nonrelativistic theory to the unique division of events space into space and time. In relativistic theory, where the division of events space into space and time is ambiguous and arbitrary, the division of the description of physical phenomena into states and equations of motion is also ambiguous and arbitrary. The state of a physical system defined at once in the entire events space corresponds far better to a unique events space, and the equations of motion enter as restrictions.

It can be shown that for its existence it makes no difference whether or not the division of the description of the physical system into states and equations of motion is carried out and just how this division is carried out. Actually, this has no meaning for the dynamics, but it is of significance for the statistics, since the statistics is understood by the calculation of the states. It is significant for the statistics precisely what is understood as the state. Statistics which correspond to different divisions of a description of a physical system into states and equations of motion lead, generally speaking, to different results.

We will describe the state of a single particle, specifying its world line in the entire events space, i.e., specifying the four functions $q^i = q^i(\tau)$, $i = 0, 1, 2,$ and 3, where τ is some parameter along the world line. Specifying /9
the momenta is not necessary because as soon as the mass $\underline{m}$ of the particle and its state, i.e., the world line, are known, the momenta p_i are determined by the re-

lationship

$$p_i = -mc \frac{g_{ik}\dot{q}^k}{\sqrt{\dot{q}^l g_{lj} \dot{q}^j}}, \qquad \dot{q}^k \equiv \frac{dq^k}{d\tau}. \tag{1.1}$$

Here

$\underline{c}$ is the speed of light and

g_{ik} is the metric tensor

$$g_{ik} = \left\| \begin{matrix} c^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{matrix} \right\|. \tag{1.2}$$

In (1.1) and everywhere in what follows the Latin indices run through the values 0, 1, 2, and 3, and the Greek indices run through the values 1, 2, and 3. Summation is carried out for repeated indices.

2. Quantum ensemble

We will assume that the state of a particle, i.e., its world line, is a random quantity. We assume for simplicity that the world line cannot reverse backwards in time, i.e., $dq^0/d\tau$ always maintains its sign. We will introduce the concept of the state density to describe random states. We will discuss at the point q^i of events the infinitely small three-dimensional area dS_k. It is evident that the number dN of world lines which cross the area dS_k is proportional to the size of the area, i.e., /10

$$dN = j^k dS_k,$$

where j^k is the proportionality coefficient. The vector j^k is proportional to the density of world lines in the neighborhood of the point q^i. $j^k(q)$ is by definition the state density at the point q. The fact that the state density is a vector should not disturb us. This is associated with the fact that one-dimensional lines are the object of the statistics and not points as in nonrelativistic statistics. Actually, in nonrelativistic statistics the state of the system is a point in phase space, but the state density is a scalar function in phase space.

It follows from the definition of j^k that the temporal component j^0 represents the average density of the particles, and the spatial components j^α represent the average flux density of the particles.

Thus the principal difference between nonrelativistic statistics and relativistic statistics consists of the fact that in the former the state density is always a scalar and in the latter it is not necessarily a scalar but can be, for example, a vector. Just the fact that the state density is a vector offers us the possibility of presenting quantum mechanics as a relativistic theory of Brownian motion. Furthermore, we will discuss only the nonrelativistic case, adopting only the relativistic concept of the state density from relativity.

We will discuss the statistical ensemble of world lines. We will assume that the nonrelativistic case occurs, i.e., the world lines deviate little from /11
some constant direction in events space. If we select this direction along the time axis, then

$$j^0 \gg |j^\alpha|, \qquad \alpha = 1, 2, 3. \tag{2.1}$$

The state of the ensemble is described by the state density $j^k(q)$. This denotes that we are discussing the statistical ensemble as some determinable physical system whose state is described by the vector j^k.

To avoid misunderstanding we note the following. Usually the concept of a statistical ensemble is introduced when the motion of an individual particle is indeterminate or the initial state of the particle is notably inaccurate. The statistical ensemble represents a set of many similar systems which occur in various states and is described by the state density. The state density represents the state of the ensemble. In the nonrelativistic case the state density W represents a function specified in phase space and is the state in the sense that, being specified at a certain instant of time, W can be uniquely defined at any succeeding instant of time. In this sense the statistical ensemble can be discussed as some determinable physical system whose state is described the function W. Thus the statistical ensemble represents a determinable system with whose help one can describe indeterminable systems. The purpose of introducing the statistical ensemble is to reduce the indeterminable physical system to a determinable one.

A different aspect occurs in the nonrelativistic case. W represents a /12
nonnegative quantity, and upon imposing normalization $W d\Omega$ can be interpreted as the probability of detecting a particle in the element of phase volume $d\Omega$. This situation in combination with the fact that W is a state permits one to speak about a random Markov process and so forth.

Generally speaking, the two indicated aspects of a statistical ensemble are independent, i.e., the state of the ensemble, being the state density, cannot be the probability density.

It is important to us in the relativity case that the statistical ensemble described by the vector j^k represents a determinable physical system. The fact that $j^k dS_k$ can be interpreted as the probability of detecting a particle in the 3-space dS_k is valid only as long as the world lines of the particles do not make zigzags in time. For relativistic particles, where the creation and destruction of pairs is possible, it is necessary to reject such an interpretation.

We postulate that in the nonrelativistic approximation (2.1) only the states j^k of the ensemble are possible which satisfy the equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial j^\alpha}{\partial q^\alpha} = 0, \tag{2.2}$$

$$\frac{\partial j^\alpha}{\partial t} + \frac{\partial}{\partial q^\beta}\left(\frac{j^\alpha j^\beta}{\rho}\right) = -\frac{\hbar^2}{4m^2}\frac{\partial}{\partial q^\beta}\left[\frac{\rho^\alpha \rho^\beta}{\rho} - \delta^{\alpha\beta}\Delta\rho\right],$$

where we introduce the symbols /13

$$\rho \equiv j^0, \quad \rho^\alpha \equiv -\frac{\partial \rho}{\partial q^\alpha}, \quad \Delta \equiv \sum_{\alpha=1}^{3} \frac{\partial^2}{\partial q^\alpha \partial q^\alpha}, \tag{2.3}$$

$\hbar$ is Planck's constant and m is the mass of the particles which form the ensemble. We will denote the statistical ensemble whose state is described by the vector j^k which satisfies the Eq. (2.2) as the quantum ensemble.

From the nonrelativistic point of view the four equations of (2.2) represent the equations of motion of the ensemble and allow us to determine from the state j^k specified at some instant of time the state at any succeeding instant of time. The equations are selected so that for certain assumptions they would be equivalent to the Schrödinger equation for a free particle [10]. The Eq. (2.2) are the vector analogue of the Einstein-Fokker-Planck equation for the motion of an ensemble of Brownian particles. We will understand the Eq. (2.2) as the nonrelativistic approximation to the equations for an ensemble of relativistic Brownian particles. The fact that relativistic concept of state density is intrinsic gives us the possibility of presenting the Schrödinger equation as the equation of motion of the statistical ensemble.

We will show [10] that if we introduce the notation

$$v^\alpha = j^\alpha / \rho, \tag{2.4}$$

where $v^\alpha = v^\alpha(q)$ is the average velocity of the particles of the ensemble at the point q and assume that the velocity has a potential /14

$$v^\alpha = \frac{1}{m}\varphi_\alpha, \quad \varphi_\alpha \equiv \partial_\alpha \varphi, \quad \partial_\alpha \equiv \partial / \partial q^\alpha, \tag{2.5}$$

then the Schrödinger equation for a nonrelativistic particle is derived from the equations (2.2). Substituting (2.4) and (2.5) into (2.2), we obtain after some transformations:

$$\frac{\partial\rho}{\partial t} + \frac{1}{m}\partial_\alpha(\rho\varphi_\alpha) = 0, \tag{2.6}$$

$$\frac{1}{m}\frac{\partial}{\partial t}(\rho\varphi_\alpha) + \partial_\beta\left(\frac{\rho\varphi_\alpha\varphi_\beta}{m}\right) = \frac{\hbar^2}{2m^2}\rho\,\partial_\alpha\left(\frac{1}{\sqrt{\rho}}\partial_\beta\partial_\beta\sqrt{\rho}\right). \tag{2.7}$$

Using (2.6) we write (2.7) in the form

$$\frac{\rho}{m}\partial_\alpha\left[\frac{\partial\varphi}{\partial t} + \frac{\varphi_\beta\varphi_\beta}{2m} - \frac{\hbar^2}{2m}\frac{1}{\sqrt{\rho}}\partial_\beta\partial_\beta\sqrt{\rho}\right] = 0. \tag{2.8}$$

We integrate (2.8). One can without limiting the generality, include the arbitrary function of the time which is derived on the right in the term $\partial\varphi/\partial\tau$, since according to (2.5) φ is defined only correct to an arbitrary additive function of the time. We have

$$\frac{\partial\varphi}{\partial t} + \frac{\varphi_\beta\varphi_\beta}{2m} - \frac{\hbar^2}{2m}\frac{1}{\sqrt{\rho}}\partial_\beta\partial_\beta\sqrt{\rho} = 0. \tag{2.9}$$

We now multiply (2.6) by $i\hbar\exp(i\varphi/\hbar)/(2\sqrt{\rho})$ and (2.9) by $-\rho\exp(i\varphi/\hbar)$ and add them. We obtain as a result the Schrödinger equation for a free particle

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$$i\hbar\frac{\partial\psi}{\partial t} + \frac{\hbar^2}{2m}\partial_\alpha\partial_\alpha\psi = 0; \tag{2.10}$$

where by ψ is denoted the quantity

$$\psi = \sqrt{\rho}\, exp(i\varphi/\hbar). \tag{2.11}$$

According to our definition of the ensemble of world lines, the quantity $j^k dS_k$ represents the average number of world lines which intersect dS_k. In the non-relativistic case it is transformed into $\rho dS_0 = \rho dV$ because of (2.1) and under the condition of normalization

$$\int\int\int_{-\infty}^{\infty} \rho\, dV = 1 \tag{2.12}$$

can be interpreted as the probability of detecting a particle in the volume dV. If the normalization condition (2.12) is fulfilled for some instant of time, then because of the first equation of (2.2) it will be fulfilled for any other instant of time.

3. Variational Principle for the Quantum Ensemble

The Eq.(2.2) for a quantum ensemble can be derived from the variational principle if one does the treatment in the form

$$S = \int L d^4 q, \qquad d^4 q = dq^0 dq^1 dq^2 dq^3, \tag{3.1}$$

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$$L = \frac{m j^\alpha j^\alpha}{2\rho} - \frac{\hbar^2}{8m}\frac{1}{\rho}\frac{\partial\rho}{\partial q^\alpha}\frac{\partial\rho}{\partial q^\alpha} + \mu(\rho - j^0). \tag{3.2}$$

Here the vector $j^k = \{j^0, j\alpha\}$ ($\rho = j^0$) represents the state density of the ensemble and j^k are the well-known functions of the variables ξ_i, m is the mass of the particles of the ensemble, and μ is introduced as a Lagrangian multiplier and represents the energy incident on the particle. The coordinates q^i in events space are independent variables. The variables $\xi = (\xi_0, \xi_1, \xi_2, \xi_3)$, ρ, and μ are subject to variation. If the quantum ensemble is considered as some liquid, then ξ_1, ξ_2, and ξ_3 represent the Lagrangian coordinates of the elements of this liquid, i.e., the set ξ_1, ξ_2, and ξ_3 represents the "number" of the element of the liquid. ξ_0 is the Lagrangian time coordinate. However it is fictitious, and L does not depend on ξ_0. The variables ξ_i enter L through the state density j^k, and

$$j^k = \frac{\partial J}{\partial \xi_{0,k}}, \qquad \xi_{i,k} \equiv \partial \xi_i / \partial q^k, \tag{3.3}$$

where

$$J = \frac{\partial(\xi_0, \xi_1, \xi_2, \xi_3)}{\partial(q^0, q^1, q^2, q^3)}. \tag{3.4}$$

We will verigy that the connection between the Lagrangian coordinates ξ_i of the quantum ensemble and the state density j^k is given by the relationships (3.3). It follows from (3.3) that

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$$j^0 dV = \frac{\partial(\xi_1, \xi_2, \xi_3)}{\partial(q^1, q^2, q^3)} dq^1 dq^2 dq^3 = d\xi_1 d\xi_2 d\xi_3, \tag{3.5}$$

i.e., selecting the Lagrangian coordinates ξ_1, ξ_2, and ξ_3 so that $d\xi_1$, $d\xi_2$, $d\xi_3$ would represent the average number of particles in the volume, where the variables ξ_α vary within the limits $\Xi_\alpha < \xi_\alpha < \Xi_\alpha + d\xi_\alpha$, $\alpha = 1, 2, 3$, we derive that j^0 is the average density of the number of particles. Furthermore,

$$j^{\alpha} = \frac{\partial(q^{\alpha}, \xi_1, \xi_2, \xi_3)}{\partial(q^0, q^1, q^2, q^3)} = \frac{\partial(q^{\alpha}, \xi_1, \xi_2, \xi_3)}{\partial(q^0, \xi_1, \xi_2, \xi_3)} \cdot \frac{\partial(q^0, \xi_1, \xi_2, \xi_3)}{\partial(q^0, q^1, q^2, q^3)} = j^0 \frac{dq^{\alpha}}{\partial t}\bigg|_{\xi_\beta = const} \tag{3.6}$$

It follows from (3.6) that j^{α} is the derivative of the density of the number of particles multiplied by the average velocity $v^{\alpha} = dq^{\alpha}/dt$. This agrees with the concept of j^{α} as the density of the flux of particles. To avoid misunderstanding we note that the Lagrangian coordinates ξ_{α} describe not the motion of the individual particles of the ensemble but the average motion of the particles of the ensemble, similar to the situation in gas dynamics where the Lagrangian coordinates describe the motion of a volume element of the gas which contains many molecules, i.e., they describe the average motion of the molecules of the gas. Thus the motion of an individual molecule of the gas is chaotic and does not correspond with this average motion.

Now it is easy to understand the meaning of the individual terms in the Lagrangian (3.2). The first term represents the density of the kinetic energy of the ensemble $m\rho v^{\alpha} v^{\alpha}/2$. The second term represents the density of the internal (potential) energy of the ensemble taken with the opposite sign. This energy /18
is associated with the chaotic motion of the ensemble's particles. Evidently this is not the total energy of chaotic motion but only that part of it which appears in the motion of the ensemble. The second term is proportional to the square of the gradient of the density ρ. This is understandable because the effect of the chaotic motion on the ordered motion appears as the diffusion which appears in the presence of a density gradient. This fact is well known from the theory of Brownian motion. Finally, the last term represents the means of introducing the symbol

$$\rho = j^0 \tag{3.7}$$

with the help of the Lagrangian multiplier μ [14]. After introducing the notation of (3.7) the Lagrangian (3.2) contains only the variables ξ_i, ρ, and μ subject to variation and their first derivatives. This simplifies the calculations.

We derive the equations of the quantum ensemble, varying (3.1) with respect to μ, ρ, and ξ_i and equating the variational derivatives to zero. The variation with respect to μ gives the relation (3.7). Furthermore,

$$\frac{\delta S}{\delta \rho} = -\frac{m j^{\alpha} j^{\alpha}}{2\rho^2} + \frac{\hbar^2}{8m}\left[\frac{\rho_\alpha \rho_\alpha}{\rho^2} + 2\frac{\partial}{\partial q^\alpha}\left(\frac{\rho_\alpha}{\rho}\right)\right] + \mu = 0, \qquad (3.8)$$

where $\rho_\alpha \equiv \partial_\alpha \rho$,

$$\frac{\delta S}{\delta \xi_i} = -\frac{\partial}{\partial q^\ell}\left[\frac{m j^\alpha}{\rho}\frac{\partial^2 y}{\partial \xi_{0,\alpha}\,\partial \xi_{i,\ell}} - \mu \frac{\partial^2 y}{\partial \xi_{0,0}\,\partial \xi_{i,\ell}}\right] = 0. \qquad (3.9)$$

Using the identity /19

$$\frac{\partial}{\partial q^\ell}\,\frac{\partial^2 y}{\partial \xi_{0,\kappa}\,\partial \xi_{i,\ell}} = 0, \qquad (3.11)$$

we remove $\partial^2 y/\partial\xi_{0,k}\,\partial\xi_{i,\delta}$ from under the derivative sign, multiply (3.9) by $\xi_{i,\beta}$, and some over i. Because of the identity

$$\xi_{i,s}\,\frac{\partial^2 y}{\partial \xi_{0,\kappa}\,\partial \xi_{i,\ell}} = \delta_s^\ell\,\frac{\partial y}{\partial \xi_{0,\kappa}} - \delta_s^\kappa\,\frac{\partial y}{\partial \xi_{0,\ell}} \qquad (3.12)$$

we obtain

$$\xi_{i,\beta}\frac{\delta S}{\delta \xi_i} = -m j^{\alpha}\partial_{\beta}\left(\frac{j^{\alpha}}{\rho}\right) + m j^{\ell}\frac{\partial}{\partial q^{\ell}}\left(\frac{j^{\beta}}{\rho}\right) + j^{0}\frac{\partial \mu}{\partial q^{\beta}} = 0. \qquad (3.13)$$

We use the identity

$$\frac{\partial}{\partial q^{\kappa}}\,\frac{\partial \gamma}{\partial \xi_{0,\kappa}} = \frac{\partial j^{\kappa}}{\partial q^{\kappa}} = 0. \qquad (3.14)$$

Introducing the notation $t = q^0$, we obtain by virtue of (3.7)

$$\frac{\partial \rho}{\partial t} + \frac{\partial j^{\alpha}}{\partial q^{\alpha}} = 0, \qquad (3.15)$$

i.e., the first equation of (2.2). Substituting $\partial\rho/\partial t$ from (3.15) into (3.13), we obtain

$$m\left\{\frac{\partial j^{\beta}}{\partial t} + \partial_{\alpha}\left(\frac{j^{\alpha} j^{\beta}}{\rho}\right)\right\} = -\rho\frac{\partial}{\partial q^{\beta}}\left(\mu - \frac{m}{2}\frac{j^{\alpha} j^{\alpha}}{\rho^{2}}\right). \qquad (3.16)$$

It follows from (3.8) that /20

$$\mu = \frac{m}{2}\frac{j^{\alpha} j^{\alpha}}{\rho^{2}} - \frac{\hbar^{2}}{2m}\frac{1}{\sqrt{\rho}}\,\partial_{\alpha}\partial_{\alpha}\sqrt{\rho}. \qquad (3.17)$$

Substituting this equation into (3.16), we obtain after transformations of the right-hand side the second equation of (2.2). Thus the Eq.(2.2) for the quantum ensemble can be derived from the variational principle.

The existence of the variational principle is very convenient in this connection, which permits the easy introduction of the concepts of the energy and momentum of the ensemble. We will calculate for this purpose the canonical energy-momentum tensor T^i_k with the help of the relation

$$T^i_\kappa = \sum_\gamma \frac{\partial L}{\partial u_{\gamma,i}} u_{\gamma,\kappa} - \delta^i_\kappa L, \qquad (3.18)$$

$$u_{\gamma,i} \equiv \partial_i u_\gamma .$$

Here the variables ξ_i, μ, and ρ, with respect to which the variation is carried out, are denoted by μ_γ. The summation is carried out over all these variables. Calculation gives

$$T^0_0 = \frac{m j^\alpha j^\alpha}{2\rho} + \frac{\hbar^2}{8m}\frac{\rho_\alpha \rho_\alpha}{\rho} = \mu\rho + \frac{\hbar^2}{4m}\partial_\alpha \partial_\alpha \rho, \qquad (3.19)$$

$$T^0_\beta = -m j^\beta = m j_\beta, \qquad j_\beta = g_{\beta\kappa} j^\kappa . \qquad (3.20)$$

for the components T^0_0 and T^0_β. Here j_β are the covariant components of the vector j^k, which are derived from j^k by dropping the indices with the help of the metric tensor (1.2). We identify T^0_0 with the energy density of the quantum ensemble and T^0_β with the momentum density. The energy E and the momentum P_β of the ensemble are defined as integrals of the quantities T^0_0 and T^0_β: /21

$$E = \int \left(\frac{m j^{\alpha} j^{\alpha}}{2\rho} + \frac{\hbar^2}{8m} \frac{\rho_{\alpha} \rho_{\alpha}}{\rho} \right) dV, \tag{3.21}$$

$$\mathcal{P}_{\beta} = - \int m j^{\beta} dV. \tag{3.22}$$

Similarly, one can define the density of the moment of the ensemble's momentum $M^{0,\alpha\beta}$ [15]. All the fields are scalar in our case, because the spin moment of the momentum is lacking. We have

$$M^{0,\alpha\beta} = q^{\beta} T^{\alpha 0} - q^{\alpha} T^{\beta 0} = m \left(q^{\beta} j^{\alpha} - q^{\alpha} j^{\beta} \right). \tag{3.23}$$

The moment of the momentum $M^{\alpha\beta}$ is derived as the integral over the volume from (3.23)

$$M^{\alpha\beta} = \int m \left(q^{\beta} j^{\alpha} - q^{\alpha} j^{\beta} \right) dV. \tag{3.24}$$

We will discuss the question of what the physical quantities introduced denote: the energy, the momentum, and the moment of momentum as applied to the statistical ensemble. For brevity we will speak only about the energy, keeping in mind the fact that everything stated about the energy is also valid for the other physical quantities. /22

From a formal point of view the state of the ensemble is defined by the set of quantities ξ_i, μ, and ρ or by the vector j^k. If j^k is specified at some instant of time, then we are able to calculate j^k with the help of the Eq.(2.2) at another instant of time. In short, the description of the statistical ensemble from a formal point of view is similar to a description of a determinable system, for example, a system consisting of a moving liquid, and a canonical method exists for defining the energy and the other physical quantities for such a system. The energy introduced by us is a function of the ensemble's state and is conserved in time, i.e., formally it can be discussed as the energy of a physical system, the so-called quantum statistical ensemble.

As has already been stated, the description with the help of the statistical ensemble is a method for the statistical description of nondeterminable systems. We will discuss, for example, a nonrelativistic Brownian particle. If at the instant of time $t = 0$ the position of the Brownian particle and its velocity are accurately known, then it is already impossible at the instant of time $t > 0$ to uniquely define its position. The reason for this is the unpredictable effect on the particle by the medium in which it is moving. One can, however, repeat very many times one and the same experiment, or, what amounts to the same thing, consider many similar systems at once which consist of a single Brownian particle which at the initial time is located in one and the same state. This /23
set of identical systems forms a statistical ensemble. The motion of the Brownian particle in the various systems of the ensemble will occur in various ways. Although the motion of each individual particle is unpredictable, it appears possible to predict the relative number of Brownian particles which will be found in this or the other state. The number of Brownian particles found in the various states represents the ensemble's state. Knowing it at one instant of time, one can predict the ensemble's state (i.e., the number of Browian particles in the various states) at any succeeding instant of time.

The ensemble's energy is the sum of the energies of the Brownian particles which constitute the ensemble. The energy is a random quantity. In the case of a very large number of particles in the ensemble the fluctuation of the ensemble's energy is much smaller than the energy itself. Therefore

$$E = N\left(\mathcal{E} + o(1/N)\right), \quad \lim_{N\to\infty} o(1/N) = 0, \tag{3.25}$$

where E is the ensemble's energy, N is the number of particles in the ensemble, and ε is the average energy of a Brownian particle. Thus the energy of the statistical ensemble (as $N \to \infty$) differs from the average energy of a Brownian particle by only a constant multiplier. In particular, if $N \to \infty$ and the ensemble's state is normalized by the relation (2.12), the ensemble's energy corresponds with the average energy of the particles. Everything that has been said about an ensemble of relativistic Brownian particles is applicable to a quantum statistical ensemble, with the difference that in the latter case the ensemble's state is described by the vector j^k.

Thus upon normalization (2.12), E, P_β, and $M^{\alpha\beta}$ from (3.21), (3.22), and (3.24) represent, respectively, the average energy, the average momentum, and the average moment of the momentum of a quantum particle. The energy density T_0^0 of the ensemble represents, in agreement with what has been stated above, the average energy of those particles which are found in the vicinity of the point q multiplied by ρ, and so forth. /24

4. Statistical Characteristics of a Quantum Particle. Correspondence Principle. Uncertainty Principle.

According to our assumptions, the behavior of a quantum particle is unpredictable. Only a statistical description of its behavior is possible. The behavior of a statistical ensemble of such particles is described by the vector j^k, and $j^k dS_k$ represents the average flux of world lines through the 3-space $dS_k = \{dV, 0, 0, 0\}$. The direction of the vector $j^k(q)$ indicates the average direction of the world lines in events space at the point q; therefore the average velocity of the ensemble's particles is

$$\langle v^\alpha \rangle = \int_{V_0} \frac{j^\alpha}{\rho} \rho \, dV = \int_{V_0} j^\alpha dV. \qquad (4.1)$$

Here V_0 denotes regular space. Here and in what follows we will assume the normalization condition (2.12) to be fulfilled. Upon this normalization $\rho(q)dV$ represents the probability of detecting a particle in the volume dV; therefore the average value $\langle F \rangle$ of the quantity F, which represents a function of the spatial coordinates $\vec{q}$, is given by the equation /25

$$\langle F \rangle = \int_{V_0} \rho F(\vec{q})\, dV. \qquad (4.2)$$

The angular brackets denote an average over the ensemble.

Furthermore, we will define the particle's average energy, average momentum, and average moment of the momentum. They are specified, as we have seen, by the relationships (3.21), (3.22), and (3.24). We will assume that the conditions (2.5) are fulfilled and that $\rho(q)$ falls off sufficiently rapidly at the spatial infinity. The latter is necessary to fulfill the condition (2.12). We introduce the operator

$$\hat{p}_\alpha = -i\hbar\, \partial/\partial q^\alpha, \qquad \hat{p}^\alpha = i\hbar\, \partial/\partial q^\alpha. \qquad (4.3)$$

Then, using the notation of (2.11), one can represent the average energy, momentum, and moment of the momentum, respectively, in the form:

$$\langle E \rangle = \int_{V_0} \left(\frac{m v^\alpha v^\alpha}{2} + \frac{\hbar^2}{8m} \frac{\rho_\alpha \rho_\alpha}{\rho^2} \right) \rho\, dV = \int \Psi^+ \frac{\hat{p}_\alpha \hat{p}_\alpha}{2m} \Psi\, dV, \qquad (4.4)$$

$$\langle \mathcal{P}_\beta \rangle = \int m v_\beta \rho\, dV = -\int \Psi^+ \hat{p}_\beta \Psi\, dV, \qquad (4.5)$$

$$\langle M^{\alpha\beta}\rangle = \int m(q^\beta v^\alpha - q^\alpha v^\beta)\rho\, dV = \int \psi^+(q^\alpha \hat{p}^\beta - q^\beta \hat{p}^\alpha)\psi\, dV, \qquad (4.6)$$

where the complex conjugate of ψ is denoted by ψ^+.

Concerning the canonical momentum p_i, which is defined by the relation (1.1), the quantum statistical ensemble does not contain information about the distribution of the particles with respect to momenta. All that we know is the average value of the momentum

$$\langle p_\alpha\rangle = -\langle m v_\alpha\rangle = \int \psi^+ \hat{p}_\alpha \psi\, dV. \qquad (4.7)$$

The mean square of the momentum can be defined, assuming that the relation

$$\langle E\rangle = \langle \frac{p_\alpha p_\alpha}{2m}\rangle . \qquad (4.8)$$

occurs. Such an assumption is natural, since systematic external forces do not act on the particle, but only a stochastic interaction with the medium (the ether). It follows from (4.8) and (4.4) that

$$\langle p_\alpha p_\alpha\rangle = \int \psi^+ \hat{p}_\alpha \hat{p}_\alpha \psi\, dV. \qquad (4.9)$$

We can draw the following conclusion from (4.2), (4.4)-(4.6), and (4.9).

It is necessary to form the operator

$$\hat{F} = F(q^{\alpha}, \hat{p}_{\beta}) \tag{4.10}$$

and calculate the quantity /27

$$< F > = \int \psi^{*} \hat{F} \psi \, dV. \tag{4.11}$$

in order to define the average value of some function $F(\vec{q},\vec{p})$ of the coordinates $\vec{q}$ and the momenta p_{α}. The formulated rule is valid for the quantities listed above: the momenta, energies, and the moment of the momentum and an arbitrary function of the coordinates, i.e., for $F(\vec{q}, \vec{p})$ containing p_{α} to no higher than the second degree.

Extrapolating this rule to arbitrary functions $F(\vec{q}, \vec{p})$, we find that it is accepted to call it the correspondence principle. Meanwhile, the validity of such an extrapolation is by no means evident. Essentially formal considerations and not physical ones serve as the basis for such an extrapolation. Moreover, using successively the correspondence principle [4], it is possible to arrive at the conclusion that there does not exist a distribution function over the coordinates and the momenta simultaneously but distribution functions exist only with respect to the coordinates or only with respect to the momenta. Actually, using the correspondence principle and defining in some manner the order of the operators in the calculation of averages of the type $< \vec{q}^{n} \vec{p}^{m} >$, it is then possible to set up a distribution function $W(\vec{q},\vec{p})$ for the momenta which appears [4], generally speaking, to be complex and at any rate is not positive. The conclusion about the absence of a distribution function for the coordinates and the momenta follows from this. This indicates that it is impossible to speak about world lines of particles and contradicts our basic position that quantum mechanics is a statistical ensemble of relativistic Brownian particles.

From our point of view the fact that the quantum ensemble does not contain /28 total information about the momenta distribution does not indicate that a suitable distribution does not exist. As an example, we introduce the case of ordinary nonrelativistic Brownian particles. It is sufficient for a description of their motion to know the distribution with respect to the coordinates, but this does not indicate that there simultaneously does not exist a distribution of the Brownian particles with respect to the momenta. It exists and is Maxwellian. Thus a boundedness of the applicability of the correspondence principle follows from our conception.

The principle of the superposition of states in quantum mechanics and the linearity of the equations of quantum mechanics are fundamental in the usual interpretation of quantum mechanics ([1], p. 26). It is not possible to imagine how one can construct quantum mechanics in the regular way, having rejected the principle of linear superposition. The entire apparatus of nonrelativistic quantum mechanics and relativistic quantum field theory are constructed on the principle of linear superposition.

In a discussion of quantum mechanics as a relativistic statistical ensemble the linearity of Schrödinger's Eq.(2.10), which is derived from (2.2) as a result of the restrictions (2.5) and the substitution of the variables (2.11), emerges as something accidental and non-fundamental. The method of calculating average values and the meaning of all the quantities is defined by the concept of a statistical ensemble and does not depend on whether or not the equations which are derived from the Eq.(2.2) are linear. If, for example, the restrictions (2.5) are abandoned, then it will not be possible, generally speaking, to derive a linear equation of the Schrödinger type. Of course, this is less convenient /29
because linear equations are easier to solve. However, in principle this does not introduce any kind of disadvantage from the point of view of the logical structure of the theory because the linearity of Eq.(2.10) has nowhere been used by us.

Thus from our point of view the principle of superposition is not a necessary feature of quantum theory. It is completely permissible that in the relativistic case it is simply not applicable and is valid only in the nonrelativistic approximation.

We will now discuss the uncertainty relationship. For a coordinate q^1 and a momentum p_1 it can be written in the form

$$\Delta p_1 \Delta q^1 \geq \hbar, \tag{4.12}$$

where

$$\Delta p_1 = \sqrt{\langle p_1^2 \rangle - \langle p_1 \rangle^2}, \qquad \Delta q^1 = \sqrt{\langle (q^1)^2 \rangle - \langle q^1 \rangle^2}. \tag{4.13}$$

It is also written similarly for the other components of q^α, p_α. From a formal point of view (4.12) is the mathematical consequence of the definition (4.3) of the momentum operator $\hat{p}_\alpha$ and the rule (4.11) for defining averages. It is a common feature of the momentum operator $\hat{p}_\alpha$ and the momentum p_i defined by the relationship (1.1) that they have identical average values $\langle p_\alpha \rangle$ and $\langle \vec{p}^2 \rangle = \langle p_\alpha p_\alpha \rangle$. In addition it is permissible that they have identical average values /30 $\langle p_\alpha p_\beta \rangle$ for $\alpha, \beta = 1, 2, 3$. Then the uncertainty relationship (4.12) is related to the momenta (1.1) of the particles comprising the quantum ensemble. The latter assumption is arbitrary and does not follow directly from our conception of a quantum statistical ensemble.

The uncertainty relationship states that the product of the uncertainties in a coordinate and the corresponding momentum for any ensemble is not less than $\hbar$ and no ensemble exists for which a coordinate and its conjugate momentum are simultaneously known exactly. This is valid both for the usual interpretation of quantum mechanics ([2], p. 172) and for our conception. However, the nature of the relationship is different in these two cases. In the case of the Copenhagen interpretation the impossibility of a particle having a specific coordinate and momentum is associated with the effect of the measuring device. For example, if the coordinate is measured exactly, then the momentum is in doubt in an unpredictable way. The question of what the momentum "really" is for a specified coordinate is an empty one since it cannot be verified by experiment.

In our conception the uncertainty relationship (4.12) exists regardless of the measurement process. It is caused by the fact that the energy of a quantum ensemble becomes large if one localizes it in a small volume $(\Delta q)^3$. Actually, according to (4.4) and (4.8),

$$\langle \vec{p}^{\,2} \rangle - \langle \vec{p} \rangle^2 = 2m \langle E \rangle - \langle \vec{p} \rangle^2 = \frac{\hbar^2}{4} \int \frac{\rho_\alpha \rho_\alpha}{\rho} dV. \qquad (4.14)$$

Since ρ is different from zero only in the volume $(\Delta q)^3$, then $|\rho_\alpha| \approx 2\rho/\Delta q$ and /31

$$(\Delta p)^2 \approx \frac{\hbar^2}{(\Delta q)^2}. \qquad (4.16)$$

Thus from our point of view the uncertainty principle is a result of the fact that every quantum ensemble, even a stationary one, possesses energy. This energy is caused by the chaotic motion of the quantum Brownian particles and the more strongly the ensemble is localized, the larger it is. From the point of view of the usual interpretation of quantum mechanics, everything is just the reverse. The uncertainty principle is primary. Its existence is caused by the properties of the measuring devices. The fact that every quantum system in the ground state possesses an energy different from zero is considered as a manifestation of the uncertainty principle.

It may appear that all this amounts to unnecessary subtleties. In the final analysis what difference does it make what is considered primary, the uncertainty principle or the presence of energy in a system in a ground state? In fact these are associated phenomena. There is, however, a difference. It appears when we pass over to the relativistic theory. Then either the uncertainty principle with the correspondence principle and the superposition principle or the concept of a statistical ensemble of relativistic Brownian particles is set as the foundation. In these two cases we arrive at two different relativistic theories which, generally speaking, do not agree with one another.

Finally, the characteristic of a measuring device to disturb the measured object so that the simultaneous measurement of a coordinate and the momentum appears impossible is produced, from our point of view, by the properties of the quantum ensemble. The properties of the latter are caused by stochastic interaction with the ether and therefore have a general nature, i.e., are valid for ensembles of any particles. In this sense the "thermal motion of the ether" is an obstacle to exact measurements similar to how under normal conditions thermal motion prevents, for example, the exact measurement of the small systematic oscillations of a pendulum. /32

5. Nonrelativistic Quantum Ensemble in an Electromagnetic Field. Stationary States of the Ensemble

We will now discuss a quantum ensemble of particles moving in a specified electromagnetic field. To take into account the interaction of the particles of the quantum ensemble with the electromagnetic field we add the term

$$L_{e\gamma} = \frac{e}{c}\rho A_0 + \frac{e}{c} j^{\alpha} A_{\alpha} . \tag{5.1}$$

to the Lagrangian (3.2). Here A_i is the 4-potential of the electromagnetic field and e is the charge of the particles forming the ensemble. We denote by $S_{e\gamma}$ the contribution to the effect which the term $L_{e\gamma}$ gives. Carrying out operations similar to those which were carried out in Section 3, we obtain

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$$\xi_{i,\beta}\frac{\delta S_{e\gamma}}{\delta \xi_i} = -\frac{e}{c} j^{\alpha}\partial_{\beta} A_{\alpha} + \frac{e}{c} j^{\ell}\partial_{\ell} A_{\beta} , \tag{5.2}$$

$$\frac{\delta S_{e\gamma}}{\delta \rho} = \frac{e}{c} A_0 . \tag{5.3}$$

Taking into account the interaction with the electromagnetic field results in the additional terms (5.2) and (5.3) in the Eq.(3.13) and (3.8), respectively. As a result we obtain in place of (3.16) and (3.17), respectively,

$$m\left\{\frac{\partial j^{\beta}}{\partial t}+\partial_{\alpha}\left(\frac{j^{\alpha}j^{\beta}}{\rho}\right)\right\}=-\rho\,\partial_{\beta}\left(\mu-\frac{m}{2}\frac{j^{\alpha}j^{\alpha}}{\rho^{2}}\right)-\frac{e}{c}j^{\alpha}F_{\alpha\beta}-\frac{e}{c}\rho\frac{\partial A_{\beta}}{\partial t}, \tag{5.4}$$

$$\mu=\frac{m}{2}\frac{j^{\alpha}j^{\alpha}}{\rho^{2}}-\frac{\hbar^{2}}{2m}\frac{1}{\sqrt{\rho}}\partial_{\alpha}\partial_{\alpha}\sqrt{\rho}-\frac{e}{c}A_{0}. \tag{5.5}$$

Here the tensor of the electromagnetic field

$$F_{ik}=\frac{\partial A_{k}}{\partial q^{i}}-\frac{\partial A_{i}}{\partial q^{k}} \tag{5.6}$$

is denoted by F_{ik}. We substitute (5.5) into (5.4). We obtain

$$m\left\{\frac{\partial j^{\beta}}{\partial t}+\frac{\partial}{\partial q^{\alpha}}\left(\frac{j^{\alpha}j^{\beta}}{\rho}\right)\right\}=-\frac{\hbar^{2}}{4m}\frac{\partial}{\partial q^{\alpha}}\left(\frac{\rho_{\alpha}\rho_{\beta}}{\rho}-\delta_{\alpha\beta}\partial_{\gamma}\partial_{\gamma}\rho\right)-\frac{e}{c}j^{k}F_{k\beta}, \tag{5.7}$$

i.e., (5.7) differs from Eq.(2.2) multiplied by m by a term of the form $ec^{-1}j^{k}F_{k\rho}$, which represents the Lorentz force acting on the 4-current ej^{k}. /34
In order to derive the complete system of equations it is necessary to add to (5.7) the Eq.(3.7) and (3.15).

We verify that the description of the quantum ensemble with the help of Eq.(5.7), (3.7), and (3.15) is equivalent to a description with the help of the Schrödinger equation for a spinless particle in an electromagnetic field

[11]. We will assume the potential condition (2.5), which in the case of a vector potential A_α different from zero is written as

$$\frac{j^\alpha}{\rho} = \frac{1}{m}\left(\varphi_\alpha - \frac{e}{c}A_\alpha\right), \quad \varphi_\alpha \equiv \partial_\alpha \varphi. \tag{5.8}$$

to be fulfilled. Using Eq.(3.15), we rewrite Eq.(5.7) in the form

$$\begin{aligned} m\rho\left\{\frac{\partial}{\partial t}\left(\frac{j^\beta}{\rho}\right) + \frac{j^\alpha}{\rho}\frac{\partial}{\partial q^\alpha}\left(\frac{j^\beta}{\rho}\right)\right\} &= \\ = \rho\left[f_\beta - \frac{e}{c}\left(\frac{\partial A_\beta}{\partial t} - \frac{\partial A_0}{\partial q^\beta}\right) - \frac{e j^\alpha}{c\rho}\left(\frac{\partial A_\beta}{\partial q^\alpha} - \frac{\partial A_\alpha}{\partial q^\beta}\right)\right]&, \end{aligned} \tag{5.9}$$

whereby f_β is denoted the quantity

$$\begin{aligned} f_\beta &= -\frac{\hbar^2}{4m}\frac{1}{\rho}\partial_\alpha\left(\frac{\rho_\alpha \rho_\beta}{\rho} - \delta_{\alpha\beta}\partial_\gamma \rho_\gamma\right) = \\ &= \frac{\hbar^2}{2m}\partial_\beta\left(\frac{1}{\sqrt{\rho}}\partial_\alpha\partial_\alpha\sqrt{\rho}\right). \end{aligned} \tag{5.10}$$

Substituting (5.8) into (5.9) and using (5.10), we obtain after some rearrangement ∠35

$$\begin{aligned} \frac{\partial}{\partial q^\beta}\left\{\frac{\partial \varphi}{\partial t} - \frac{e}{c}A_0 + \frac{1}{2m}\left(\varphi_\alpha\varphi_\alpha - \frac{2e}{c}A_\alpha\varphi_\alpha + \frac{e^2}{c^2}A_\alpha A_\alpha\right) - \right. \\ \left. - \frac{\hbar^2}{2m}\frac{1}{\sqrt{\rho}}\partial_\alpha\partial_\alpha\sqrt{\rho}\right\} = 0. \end{aligned} \tag{5.11}$$

Integration of (5.11) gives

$$\frac{\partial \varphi}{\partial t} - \frac{e}{c} A_0 + \frac{1}{2m}\left(\varphi_\alpha \varphi_\alpha - \frac{2e}{c} A_\alpha \varphi_\alpha + \frac{e^2}{c^2} A_\alpha A_\alpha\right) - \frac{\hbar^2}{2m} \frac{1}{\sqrt{\rho}} \partial_\alpha \partial_\alpha \sqrt{\rho} = 0. \tag{5.12}$$

We set the arbitrary function of time obtained upon the integration equal to zero, since without limiting the generality it can be included in the term $\partial\varphi/\partial t$, because φ is determined with accuracy only to an additive function of the time.

The substitution of (5.8) into (3.15) with subsequent multiplication by $(2\sqrt{\rho})^{-1}$ gives

$$\frac{\partial \sqrt{\rho}}{\partial t} + \frac{1}{m} \partial_\alpha \sqrt{\rho}\left(\varphi_\alpha - \frac{e}{c} A_\alpha\right) + \frac{\sqrt{\rho}}{2m} \partial_\alpha\left(\varphi_\alpha - \frac{e}{c} A_\alpha\right) = 0. \tag{5.13}$$

We now multiply (5.12) by $-\sqrt{\rho}\exp(i\varphi/\hbar)$ and (5.13) by $i\hbar\exp(i\varphi/\hbar)$ and add them. We obtain a single complex equation

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$$i\hbar \frac{\partial \psi}{\partial t} - \frac{e}{c} A_0 \psi - \frac{1}{2m}\left(-i\hbar \frac{\partial}{\partial q^\alpha} - \frac{e}{c} A_\alpha\right)\left(-i\hbar \frac{\partial}{\partial q^\alpha} - \frac{e}{c} A_\alpha\right)\psi = 0, \tag{5.14}$$

where ψ is given by the Eq.(2.12). Eq.(5.14) represents the Schrödinger equation (2.12) for a spinless particle in an electromagnetic field.

We will now discuss the problem of finding the stationary states of the ensemble in a steady electromagnetic field. Such a problem is of interest since it is easy to observe stationary ensembles. The steady-state condition of the of the electromagnetic field is written

$$\frac{\partial F_{ik}}{\partial t} = 0. \tag{5.15}$$

In this case the electric field E_α is a potential one, since

$$\operatorname{rot} \vec{E} = \frac{1}{c} \frac{\partial \vec{H}}{\partial t} = 0. \tag{5.16}$$

Therefore it is possible to set

$$E_\alpha = \frac{1}{c} \partial_\alpha A_0 , \tag{5.17}$$

where A_0 does not depend on the time. The possibility of the selection of (5.17) by virtue of the definition of the electric field

$$E_\alpha = \frac{1}{c} F_{\alpha 0} = \frac{1}{c} \partial_\alpha A_0 - \frac{1}{c} \frac{\partial A_\alpha}{\partial t} \tag{5.18}$$

implies that /37

$$\frac{\partial A_\alpha}{\partial t} = 0. \tag{5.19}$$

Thus to have a steady electromagnetic field the 4-potential A_i can always be selected to be independent of the time. We will assume that it is selected precisely in this way.

We will seek steady states of the ensemble, i.e., states which satisfy the condition

$$\frac{\partial f^{\alpha}}{\partial t} = 0. \tag{5.20}$$

This indicates that we should set all the time derivatives in Eq.(5.7) and (3.15) equal to zero. Repeating with the condition (5.20) all the operations which led us from (5.7) and (3.15) to the Eq.(5.12) and (5.13), we obtain in place of (5.12) and (5.13), respectively,

$$\begin{aligned} &\frac{e}{c}A_0 + \frac{1}{2m}\left(\varphi_\alpha \varphi_\alpha - \frac{2e}{c}\varphi_\alpha A_\alpha + \frac{e^2}{c^2}A_\alpha A_\alpha\right) - \\ &\qquad - \frac{\hbar^2}{2m}\frac{1}{\sqrt{\rho}}\partial_\alpha \partial_\alpha \sqrt{\rho} = E(t), \end{aligned} \tag{5.21}$$

$$\frac{1}{m}\frac{\partial\sqrt{\rho}}{\partial q^\alpha}\left(\varphi_\alpha - \frac{e}{c}A_\alpha\right) + \frac{\sqrt{\rho}}{2m}\partial_\alpha\left(\varphi_\alpha - \frac{e}{c}A_\alpha\right) = 0. \tag{5.22}$$

Here E(t) is an arbitrary function of the time which arises upon integration of equation (5.11) with $\partial\varphi/\partial t = 0$. Since the left-hand side of Eq.(5.21) does /38
not depend on the time, the function E(t) can only be constant. We multiply (5.21) by $-\sqrt{\rho}\,\exp(i\varphi/\hbar)$ and (5.22) by $i\hbar\,\exp(i\varphi/\hbar)$ and add them. We obtain the equation

$$\frac{1}{2m}\left(-i\hbar\partial_\alpha - \frac{e}{c}A_\alpha\right)\left(-i\hbar\partial_\alpha - \frac{e}{c}A_\alpha\right)\psi - \frac{e}{c}A_0\psi = E\psi, \tag{5.23}$$

where E is a real constant and ψ is a complex function (2.11). It is necessary to require that the functions fall off sufficiently rapidly at infinity in order that the condition

$$\int \psi^{+} \psi \, dV = \int \rho \, dV = 1. \qquad (5.24)$$

be fulfilled. This requirement follows from the concept of ρdV as the probability of detecting a particle in the volume dV.

Thus the search for stationary states of the ensemble has led us to the problem of eigenvalues. We note that there corresponds, generally speaking, to a stationary state of the ensemble a wave function ψ which depends on the time and is consequently nonstationary. We bear the following in mind. We will find a function $\psi = \psi_E (\vec{q})$ which is a solution of Eq.(5.23) and (5.24) and then seek a solution of the Eq.(5.14), with the same stationary A_i, in the form $\psi = \psi(\vec{q},t)$ with the initial condition

$$\psi(\vec{q},0) = \psi_E(\vec{q}). \qquad (5.25)$$

The solution has the form /39

$$\psi(\vec{q},t) = e^{-\frac{iEt}{\hbar}} \psi_E(\vec{q}). \qquad (5.26)$$

The function ψ depends on the time and consequently, is nonstationary, although it describes a stationary state, and all physical quantities are steady-state. Physicists have long ago become accustomed to this and assume that it should be so.

However, if one reflects on this fact, then it seems strange. The impression is created that the description with the help of a wave function is some artificial, i.e., not corresponding to the nature of things. As a result such an artificial description ceases at a certain degree of development of the theory to be an adequate description of reality.

Conclusions

And so we have shown that nonrelativistic quantum mechanics (at least for one particle) can be understood from the point of view of relativistic classical mechanics as some theory of relativistic Brownian motion. The expression "Brownian motion" indicates only that the motion of the particle is random and unpredictable. The particle has a world line which is random, but the motion of an ensemble of particles is determinable. The stochastic nature of the motion of the particles can be understood as the result of its interaction with the medium (ether). However, it is possible to understand it somehow differently or, in /40
general, not to understand it. At a given stage this has no meaning. It is significant that even nonrelativistic quantum mechanics is fundamental to the relativistic theory and can be understood only from the relativistic standpoint. From our point of view quantum mechanics is no more than relativistic statistics; Planck's constant represents a measure of the interaction of a particle with the ether, similar to how the diffusion coefficient in the theory of Brownian motion describes the degree of the medium's effect on the particle's motion. In light of the fact that nonrelativistic quantum mechanics is already a relativistic theory, the problem of creating a relativistic quantum theory can be rejected more simply than is customarily thought.

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